Detailed Program of the Workshop

Inverse Problems: Theoretical and Numerical Aspects

Laboratoire de Mathématiques de Reims, December 17-19 2018

December 17

14h30 Opening of the Workhop

15h00-16h00 Mourad Bellassoued (Tunis)

A Borg-Levinson theorem for magnetic Schrödinger operators on a Riemannian manifold

This talk is concerned with uniqueness and stability issues for the inverse spectral problem of recovering the magnetic field and the electric potential on a Riemannian manifold from some asymptotic knowledge of the boundary spectral data of the corresponding Schrödinger operator under Dirichlet boundary conditions. The spectral data consist in some asymptotic knowledge of a subset of eigenvalues and Neumann traces of the associated eigenfunctions of the magnetic Laplacian. We also address the same question for Schrödinger operators under Neumann boundary conditions, in which case we measure the Dirichlet traces of eigenfunctions. In our results we characterize the uniqueness of the magnetic field from a rate of growth of the eigenvalues, combined with suitable asymptotic properties of boundary observation of eigenfunctions of the associated magnetic Schrödinger operator. To our best knowledge, this is the first result proving uniqueness from such general asymptotic behavior of boundary spectral data. This is a joint work with Mourad Chouli, David Dos Santos Ferreira, Yavar Kian and Plamen Stefanov.

16h00-16h30 *Coffee break*

16h30-17h30 Evgeny Korotyaev (Saint Petersburg)

Inverse spectral theory for perturbed torus

We consider the Laplacian on the rotational symmetric finite manifolds, which are periodic with respect to the axis of the manifold, i.e. we consider the Laplacian on the torus. We study inverse problems for Laplacian on the torus. Moreover, we obtain stability estimates: the spectral data in terms on the profile (the radius of rotation) and conversely, the profile in terms of the spectral data. It is a joint work with Hiroshi Isozaki.

17h30-18h30 Michel Kern (INRIA)

Numerical tools for inverse problems, I

In this set of lectures, I will give an introduction to numerical methods for inverse problems. I will concentrate on a small number of generally applicable methods, deferring to the other presentations to show how they have to be complemented by specific domain knowledge in any particular application.

In the first lecture, I will show examples of inverse problems and discuss the mean feature of inverse problems, which is their ill-posed character, or extreme lack of stability with respect to the data.

December 18

09h00-10h00 Michel Kern (INRIA)

Numerical tools for inverse problems, II

The second lecture will discuss regularization methods, mainly in the context of integral equations of the first kind (as the principal example of linear inverse problems). The Singular Value Decomposition for matrices and the Singular Value Expansion for operators, will be introduced to analyze the ill-posedness of the problems. Regularization methods can be used to recover some stability, at the expense of solution accuracy. SVD/SVE will shed some light on this compromise.

10h00-10h30 *Coffee break*

10h30-11h30 Houssem Haddar (Ecole polytechnique)

Transmission eigenvalues and other spectra associated with inverse scattering problems

Transmission eigenvalues correspond to frequencies for which there exists a (generalized) incident wave that does not scatter. This spectrum can be seen as the counter part of so-called resonances. We first review some recent methods to identify this spectrum from multistatic data. We then show how these methods can be used to identify other (artificially constructed) spectra associated with the scatterer that may have simpler structures and would be more relevant to address the inverse spectral problem.

11h30-12h30 Mourad Sini (Linz)

Acoustic Imaging Using Resonating Bubbles as Contrast Agents

It is well known that imaging using reflected waves measured away from the target is a highly unstable process. Hence targets characterized by high contrasts can, relatively, be imaged while low contrasted targets, as early stage tumors, cannot be differentiated. One way to handle such low contrasted targets is to inject small, i.e. localized, particles inside the target and measure the fields created by such perturbations. These particles will play the role of contrast agents, i.e. creating the missing contrasts.

In the literature, there are several imaging modalities based on such ideas. Depending on the imaging modalities, micro or nano scaled particles are used. In our talk, we will focus mainly on the use of micro-bubbles in the framework of acoustic imaging. The main results we will discuss are the following:

1. Injecting single bubbles at a time, we can recover the total field at the location of the bubbles from the measured backscattered farfield related to a one fixed incident direction. This can be done using arbitrary incident frequencies.

2. Injecting dimers at a time, i.e. two-by-two close bubbles, we can recover the Green's function at the centers of the two bubbles from the measured backscattered farfield at one fixed incident direction. This can be done only using nearly resonating incident frequencies. Here the resonances are the ones related to the small bubbles, i.e. Minnaert or Fabry-Perot resonances depending on the type of bubbles' contrast.

Hence, we reduce the underlying inverse medium scattering problem to the recovery of the medium (i.e. the index of refraction) form internal values of the total fields or the Green's function. To solve these problems, we provide explicit reconstruction formulas and discuss their stability against errors and possible noise.

12h30-14h30 Lunch

14h30-15h30 Florian Faucher (INRIA)

Convergence and stability of quantitative inverse problems for applications to seismic FWI

We study the inverse problem associated with the time-harmonic wave equation, with applications to the reconstruction of subsurface Earth media. We consider the propagation of waves in a domain Ω, where we define the forward problem from the displacement vector field $\boldsymbol{u}$, solution to

$$-\rho\, \omega^2\, \boldsymbol{u} - \nabla\sigma = g \quad \text{in } \Omega,$$

where g is the source, ρ the density and σ the stress tensor. The inverse problem is designed to reconstruct the physical parameters (contained in the stress tensor) of the medium. Note that in this seismic context, the only accessible data are partial backscattered data, obtained from one-side illumination (namely the Earth surface), such data naturally increase the challenge of the reconstruction procedure.

We consider quantitative reconstruction and use the Full Waveform Inversion (FWI) method, which is based upon an iterative minimization algorithm. We study the convergence of the minimization by introducing the framework of finite Curvature/Limited Deflection (FC/LD) problems. Then FC/LD properties can be obtained by restricting the model space in order to have strictly quasiconvex attainable set. We can further estimate numerically the size of the basin of attraction depending on charasteristics such as the frequency, the geometry of the search direction and the choice of parametrization.

This analysis of convergence can be related to stability estimates in order to provide a consistent scheme where frequency progression is justified from the quantitative estimates. Eventually, we illustrate our approach with isotropic elastic medium reconstructions, starting from minimal information on the models. This also serves to illustrate the numerical difficulties of the large scale optimization problem arising in geophysics. If time allows, we shall also discuss the use of specific misfit functional adapted to the recent update in measures technology, which allows the separation between observational and computational acquisitions.

15h30-16h30 Andrea Posilicano (Como)

Inverse scattering for the Laplace operator with boundary conditions on Lipschitz surfaces

We present an abstract framework for the scattering theory of the Laplace operator with self-adjoint boundary conditions on an interface. This provides a representation for the Scattering Matrix leading to a factorized formula for the corresponding Far Field operator. As applications, we consider two kinds of semi-transparent boundary conditions and show how to recover the shape of the interface from the knowledge of the scattering data at a fixed frequency. This is a joint work with Andrea Mantile.

16h30-17h00 *Coffee break*

17h00-18h00 Jérémi Dardé (Toulouse)

Inverse obstacle problem with partial Cauchy data for the Laplace equation – A shape optimization approach

We study the inverse problem of obstacle detection for the Laplace equation with partial Cauchy data with a shape optimization approach. The usual strategy is to reduce the inverse problem into the minimization of a coast-type functional, i.a. the Kohn-Vogelius functional. But due to the lack of data on some part of the boundary of the domain, this standard strategy fails. Therefore, we add the Cauchy data on the inaccessible part of the boundary as variables of our minimization problem, solving the geometrical inverse problem and the data completion problem once in a unique minimization procedure. We present some theoretical results (convergence, regularization) as well as several numerical experiences to show the performance of the algorithm. This is a joint work with Fabien Caubet and Matias Gogoy.

20h *Dinner at brasserie Excelsior*

December 19

09h00-10h00 Michel Kern (INRIA)

Numerical tools for inverse problems, III

In the last lecture, I will turn to parameter identification problems in differential equations, as an example of nonlinear inverse problems. After formulating the problem as least-square minimization for the output error, I will describe the adjoint state method to efficiently compute the gradient of the coast functional. I will also briefly discuss the questions of parametrization and the optimize vs discretize debate.

10h00-10h30 *Coffee break*

10h30-11h30 Eric Soccorsi (Aix-Marseille)

Determining the space-dependent variable order of times-fractional diffusion equations

Space-dependent anomalous diffusion processes can be described by diffusion equations with time-fractional derivatives of space-dependent variable order. This talk is concerned ith the uniqueness issue in the inverse problem of determining the space-dependent variable order coefficients by the knowledge of a suitable time-sequence of partial Dirichlet-to-Neumann maps. It is based on a joint work with Y. Klan (Marseille) and Y. Yamamoto (Tokyo).

11h30-12h30 Laurent Bourgeois (ENSTA)

An inverse obstacle problem for the wave equation in a finite time interval

In this talk we consider an inverse obstacle problem for the wave equation in a finite time interval. After a short discussion about uniqueness for this problem, we present a new approach to solve it. It consists in coupling a mixed formulation of quasi-reversibility to update the scattering solution for a given obstacle and a level set method to update the obstacle for a given solution. A few numerical examples in 2D will illustrate the feasibility of the method. This is a joint work with Dmitry Ponomarev and Jérémi Dardé.