QUALITATIVE PROPERTIES AND SUPPORT COMPACTNESS OF SOLUTIONS FOR QUASILINEAR SCHRÖDINGER EQUATION WITH SIGN CHANGING POTENTIALS

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In this work, we deal with the following general elliptic equation involving the weighted $p-$Laplace operator

$$(\mathcal{P}) \qquad \begin{cases} -\Delta_p u + V(x)u = a(x)|u|^{q-1}u & \text{in } \mathbb{R}^N \\ u \in \mathcal{D}^{1,p}(\mathbb{R}^N) \cap L^{q+1}(\mathbb{R}^N) \end{cases}$$

where $N \geq 3$, $0 < q$, $2 \leq p$ and $a(x)$, $Vx)$ change sign in $\mathbb{R}^N$ and satisfy the following assumtions:

(A) $a \in L^\infty(\mathbb{R}^N)$, $\lim\limits_{\|x\| \to +\infty} a(x) = a_\infty < 0$ and there exist $R_0 > 0$ and $y_0 \in \mathbb{R}^N$ such that

$$a(x) > 0 \quad \forall\, x \in B(y_0, R_0).$$

(V) $V \in L^\infty(\mathbb{R}^N)$, $\lim\limits_{\|x\| \to +\infty} V(x) = V_\infty > 0$.

In a first part and on one hand, we study the existence of solutions of $(\mathcal{P})$. More precisely, we show that if $2 \leq p < N$ and $0 < q < 1$, the problems admits infinitely many weak solutions. Note that for the case $p = 2$, all weak solutions of $(\mathcal{P})$ belong to $\mathcal{C}^2(\mathbb{R}^N)$. But for $p > 2$, the weak solutions of the problem are not generally in $\mathcal{C}^2(\mathbb{R}^N)$. We give two examples that deal with different cases. On the other hand, we prove that every classical solution of $(\mathcal{P})$ tends to zero at infinity and has compact supports.

In a second part, we study for $p > 2$ the existence and the behaviour of nontrivial classical radial solutions of $(\mathcal{P})$ as well as the number of their zeros in their supports. We show that the question of the existence and nonexistence of nontrivial classical radial solutions of the problem as well as the number of their zeros in their supports depends essentially on the number of spheres of $\mathbb{R}^N$ on which the potential a and V are simultaneously null. We give a classification of these classical radial solutions.

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