EXPONENTIAL DECAY FOR THE LAMÉ SYSTEM WITH FRACTIONAL TIME DELAY AND BOUNDARY FEEDBACK

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Abstract : Our interest in this work is to analyse the asymptotic behaviour of a Lamé system with an internal fractional time delay and a boundary damping of Neumann type. Assuming the weight of the delay is small enough, we show that the system is well-posed using the semigroup theory. Furthermore, we introduce a Lyapunov functional that gives the exponential decay.

Keywords : Lamé system, Internal delay, Exponential stability, Semigroup theory.

MSC2010 : 35B40, 93D15, 74D05.

Let Ω be a bounded domain in $\mathbb{R}^{\mathbf{n}}$ with smooth boundary Γ of class C^2 and we assume that $\Gamma = \Gamma_0 \cup \Gamma_1$, where Γ_0 and Γ_1 are closed subsets of Γ with $\Gamma_0 \cap \Gamma_1 = \emptyset$. The system is given by :

$$(P) \qquad \begin{cases} u_{tt} - \mu\Delta u - (\mu+\lambda)\nabla(\text{div } u) + a_1\partial_t^{\alpha,\eta}u(x,t-\tau) = 0 \quad \text{in } \Omega\times(0,+\infty), \\ u = 0 \quad \text{in } \Gamma_0\times(0,+\infty), \\ \mu\frac{\partial u}{\partial\nu} + (\mu+\lambda)(\text{div } u)\nu = -a_2 u_t(x,t) \quad \text{in } \Gamma_1\times(0,+\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x). \quad \text{in } \Omega \\ u_t(x,t-\tau) = f_0(x,t-\tau) \quad \text{in } \Omega\times(0,\tau). \end{cases}$$

where μ, λ are Lamé constants, $u = (u_1, u_2, ..., u_n)^T$. Moreover, $a_1 > 0$, $a_2 > 0$ and the constant $\tau > 0$ is the time delay. ν stands for the unit outward normal to Γ. The notation $\partial_t^{\alpha,\eta}$ stands for the generalized Caputo's fractional derivative of order α with respect to the time variable and is defined by

$$\partial_t^{\alpha,\eta}w(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha}e^{-\eta(t-s)}\frac{dw}{ds}(s),\, ds \qquad 0<\alpha<1, \qquad \eta\geq 0.$$

The main result of this work is the following.

Theorem 1 *For any $a_2 > 0$ there exist positive constants a_0, C_1, C_2 such that*

$$E(t) \leq C_1 e^{-C_2 t}E(0),$$

for any regular solution of problem (P) with $0 \leq a_1 < a_0$. The constants a_0, C_1, C_2 are independent of the initial data but they depend on a_2 and on the geometry of Ω.

References

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