

**New Results on Abstract Differential Equations of Elliptic Type  
with Nonlocal Boundary Coefficient-operator Conditions in the  
Framework of Hölderian Spaces: Noncommutative Cases**

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This work is devoted to the abstract study of operational second order differential equations of elliptic type with nonregular coefficient-operator boundary conditions in a non commutative framework. The study is performed when the second member  $f$  belong to  $C^\theta([0, 1]; X)$ , with general  $0 < \theta < 1$ ,  $X$  being a Banach space. We give some new results by using semigroups and interpolation theory. Necessary and sufficient conditions of compatibility are established to obtain the classical solution. Maximal regularity properties are also studied.

**Keywords:** Second order differential-operator equations; Nonlocal Boundary conditions; Non commutative; Analytic semigroups, Interpolation spaces; Hölder spaces.

**Introduction**

Let  $X$  be a complex Banach space. We consider the following operational second order differential equation of elliptic type

$$u''(x) + Au(x) = f(x), \quad x \in [0, 1], \quad (1)$$

together with the following nonlocal coefficient-operator boundary conditions

$$\begin{cases} u(0) = u_0 \\ u(1) + Hu'(0) = u_{1,0}, \end{cases} \quad (2)$$

where  $A$  is a closed linear operator with domain  $D(A)$  not necessarily dense in  $X$  and  $H$  is a closed linear operator with domain  $D(H)$ ,  $f \in C^\theta([0, 1]; X)$  with  $0 < \theta < 1$  and  $u_0, u_{1,0}$  are given elements of  $X$ . As we will see, equation (1) is the abstract writing of  $\Delta u = f$  for instance.

Our main goal is to seek a classical solution of the problem (1)-(2), that is a function  $u$  such that

$$\begin{cases} i) u \in C^2([0, 1]; X) \cap C([0, 1]; D(A)) \\ ii) u(0) \in D(H) \\ iii) u \text{ satisfies (1)-(2)}. \end{cases}$$

We can also seek for a classical solution  $u$  satisfying the maximal regularity property  $u'', Au \in C^\theta([0, 1]; X)$ .

## References

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